Field theoretic study of light hypernuclei

P.K. Panda
Institute of Physics, Bhubaneswar-751005, India. *

R. Sahu
Physics Department, Berhampur University, Berhampur-760007, India.
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A nonperturbative field theoretic calculation has been made for the s-shell hypernuclei. Here we dress the Λ- and Σ- hypernuclei with off-mass shell pion pairs. The analysis replaces the scalar isoscalar potential by quantum coherent states. The binding energies of $^{4+n\Lambda}$He ($n = 0, 1, 2$) agree quite well with the RMF result of Greiner. The experimental binding energies of ^{4}He and ^{3}He are reasonably well reproduced in our calculation. A satisfactory description of the relevant experimental Λ- and Σ- separation energies has been obtained.

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I. INTRODUCTION

The investigation of the possible extension of the periodic table in the sector of strangeness [1] has been a topic of current interest. Over the years, various theoretical calculations [1–9] have been carried out to understand the properties of the hypernuclei with varying success. Greiner and his group have performed the relativistic mean field (RMF) approach to study hypernuclei. The RMF has been quite successful to describe baryons interacting through mesons. This model has been used to study the binding energy of the Λ-hypernuclei, $^{4+n\Lambda}$He, $^{16+n\Lambda}$O and $^{40+n\Lambda}$Ca. They found the binding energy of these nuclei to increase when hyperons are added to normal nuclei due to the opening of new degrees of freedom. They have also studied exotic multi-strange nuclei which are formed by adding Σ's and Ξ's within the above RMF approach. Calculations have also been performed using Monte-carlo method with the potential well depth calculated variationally with fermi-hyperneted chain method [9]. However, they are unable to reproduce simultaneously the binding energies of ^{4}He and the neighbouring hypernuclei. Not many theoretical calculations exist with Σ-hyperons. Hence it would be quite intersting to develop a model which simultaneously reproduces the binding energies of ^{4}He and ^{3}He as well as the neighbouring hypernuclei (both Λ and Σ hypernuclei).

Recently we have developed a nonperturbative technique based on quantum field theory [10–12] to study light nuclei and nuclear matter. Here the σ-meson effects are simulated through isosinglet scalar cloud of pair of off-shell pions with a coherent state. This corresponds to the quantum picture of classical fields of Walecka model [13]. Such a construct also includes higher order effects [10–12]. In this model the nucleus contains, with a small probability, a finite expectation value for off mass shell scalar isosinglet pion pairs which may have observable effects. The calculation is based on the nonperturbative techniques of field theory in which the nucleons are dressed with off mass-shell pions.

This model has been successfully applied to study the binding energy and other properties of deuteron [10] and ^{4}He [11] and nuclear matter [12]. In view of the above successes, it would be quite interesting to apply the same to a study of the light hypernuclei here.

In section II, we construct the effective Hamiltonian along with a scalar isoscalar coupling of pions with the nucleons and hyperons and construct translationally invariant states explicitly with baryons and off mass shell pion cloud. In section III we calculate the energy expectation value and extremise it. We discuss the results and possible experimental signatures in sectionn IV. The conclusions drawn from the present study are given in section V.

II. THEORY

The Langrangian density for the pion, nucleon and hyperon system is taken as [10]

*Present address: Institute of Astrophysics, NAPP Group, Bangalore-560034, India
email: prafulla@iiap.ernet.in

$$
\begin{aligned}
\mathcal{L} = & \bar{N}(i\gamma^\mu\partial_\mu - M_N + G_{\pi NN}\gamma_5\phi)N + \bar{\Lambda}(i\gamma^\mu\partial_\mu - M_\Lambda + G_{\pi\Lambda\Lambda}\gamma_5\phi)\Lambda \\
& + \frac{1}{2}(\partial_\mu\phi_i\partial^\mu\phi_i - \mu^2\phi_i\phi_i)
\end{aligned} \tag{1}
$$

where M_N, M_Λ and μ are the masses of nucleon, hyperon and pion, respectively. $G_{\pi NN}$ and $G_{\pi\Lambda\Lambda}$ are the coupling constants for pion-nucleon and pion-hyperon. We shall consider equation (1) in the non-relativistic limit. Then the effective Hamiltonian density for the system with nucleons, hyperons and pions are given as [10,11]

$$
\mathcal{H}(\mathbf{z}) = \mathcal{H}_N(\mathbf{z}) + \mathcal{H}_\Lambda(\mathbf{z}) + \mathcal{H}_M(\mathbf{z}) + \mathcal{H}_I(\mathbf{z}). \tag{2}
$$

Here the nucleon Hamiltonian density $\mathcal{H}_N(\mathbf{z})$ and hyperon Hamiltonian density $\mathcal{H}_\Lambda(\mathbf{z})$ are given by

$$
\mathcal{H}_N(\mathbf{z}) = N_\alpha(\mathbf{z})^\dagger(M_N - \frac{\bigtriangledown_z^2}{2M_N})N_\alpha(\mathbf{z}), \tag{3a}
$$

$$
\mathcal{H}_\Lambda(\mathbf{z}) = \Lambda_\beta(\mathbf{z})^\dagger(M_\Lambda - \frac{\bigtriangledown_z^2}{2M_\Lambda})\Lambda_\beta(\mathbf{z}). \tag{3b}
$$

The meson Hamiltonian density, $\mathcal{H}_M(\mathbf{z})$, is given by

$$
\mathcal{H}_M(\mathbf{z}) = \frac{1}{2}[(\partial_0\phi_i(\mathbf{z}))^2 + (\bigtriangledown\phi_i(\mathbf{z}))^2 + \mu^2(\phi_i(\mathbf{z}))^2] \tag{3c}
$$

and the interaction Hamiltonian density between baryons and pions is given by

$$
\begin{aligned}
\mathcal{H}_I(\mathbf{z}) = & i\frac{G_{\pi NN}}{2M_N}N_\alpha(\mathbf{z})^\dagger(\sigma\cdot\bigtriangledown\phi(\mathbf{z}))N_\alpha(\mathbf{z}) + \frac{G_{\pi NN}^2}{2M_N}N_\alpha(\mathbf{z})^\dagger N_\alpha(\mathbf{z})\phi_i(\mathbf{z})\phi_i(\mathbf{z}) \\
& + i\frac{G_{\pi\Lambda\Lambda}}{2M_\Lambda}\Lambda_\beta(\mathbf{z})^\dagger(\sigma\cdot\bigtriangledown\phi(\mathbf{z}))\Lambda_\beta(\mathbf{z}) + \frac{G_{\pi\Lambda\Lambda}^2}{2M_\Lambda}\Lambda_\beta(\mathbf{z})^\dagger\Lambda_\beta(\mathbf{z})\phi_i(\mathbf{z})\phi_i(\mathbf{z}).
\end{aligned} \tag{3d}
$$

In addition we include repulsive term $\mathcal{H}_R(\mathbf{z})$ and Coulomb term $\mathcal{H}_C(\mathbf{z})$ in the Hamiltonian density which are discussed later. We note that here we take the ordered products for the above expressions, so that the vacuum energy is zero. In the above α stands for both isospin and spin indices, such that $\alpha = 1,2$ stands for proton with spin $= \pm 1/2$ and $\alpha = 3,4$ stands for neutron with spin $= \pm 1/2$ and β stands for hyperons. Here $N_\alpha^\dagger$ and N_α are the nucleon creation and annihilation operator. $\Lambda_\beta^\dagger$ and Λ_β are the hyperon creation and annihilation operators. The matrix ϕ is given as $\phi(\mathbf{z}) = \tau_i\phi_i(\mathbf{z})$. We expand the field operator $\phi_i(\mathbf{z})$ in terms of creation and annihilation operators of off-mass shell mesons satisfying equal time algebra as [14]

$$
\phi_i(\mathbf{z}) = \frac{1}{\sqrt{2\omega_z}}(a_i(\mathbf{z})^\dagger + a_i(\mathbf{z})). \tag{4}
$$

In the perturbative basis we have $\omega_z = (\mu^2 - \bigtriangledown_z^2)^{\frac{1}{2}}$. We shall still use this, but we note that since we shall take an arbitrary number of pions in a coherent manner as given later, the results shall be nonperturbative. Here $i = 1,2,3$ stand for isospin indices of pions. $a_i(\mathbf{z})$ and $a_i(\mathbf{z})^\dagger$ are the annihilation and creation operators of the mesons.

We shall now consider the system with n_N nucleons and n_Λ hyperons with a dressing of meson pairs. For this purpose, we shall first define the state in a manner which shall be more convenient for field theoretic calculations and then consider the energy expectation values to obtain the nucleon wavefunction and the meson dressing. Clearly, the two pion dressing [10,11] will simulate the effect of hypothetical σ-meson exchange. One objective here is to show that such a picture of nuclear structure with Walecka's model generalised to include quantum two pion condensate can be a viable alternative, and we need not take separately a σ-meson. With this in mind, we start with the notation for the form of nucleon and hyperon creation operators as

$$
N_\alpha^c(\mathbf{x})^\dagger = \int U_\alpha^N(\mathbf{x} - \mathbf{x}_\alpha)N_\alpha(\mathbf{x}_\alpha)^\dagger d\mathbf{x}_\alpha, \tag{5a}
$$

$$
\Lambda_\alpha^c(\mathbf{x})^\dagger = \int U_\alpha^\Lambda(\mathbf{x} - \mathbf{x}_\alpha)\Lambda_\alpha(\mathbf{x}_\alpha)^\dagger d\mathbf{x}_\alpha, \tag{5b}
$$

so that, the creation operator for the helium system is taken as

$$\mathcal{N}_S(\mathbf{x})^\dagger = \prod_{\alpha=1}^{n_N} N^c_\alpha(\mathbf{x})^\dagger \prod_{\beta=1}^{n_\Lambda} \Lambda^c_\beta(\mathbf{x})^\dagger. \tag{6}$$

The background meson fields are taken through a coherent state type of formalities with the meson cloud creation operator being [10,14]

$$\mathcal{M}(\mathbf{x})^\dagger = e^{B(\mathbf{x})^\dagger}, \tag{7}$$

where

$$B(\mathbf{x})^\dagger = \frac{1}{2}\int f_M\left(\mathbf{x} - \frac{\mathbf{z}_1+\mathbf{z}_2}{2}\right) f(\mathbf{z}_1-\mathbf{z}_2) a_i(\mathbf{z}_1)^\dagger a_i(\mathbf{z}_2)^\dagger d\mathbf{z}_1 d\mathbf{z}_2. \tag{8}$$

In the above, we are taking the nucleus to be centered around the point $\mathbf{x}$, and so also the mesons. The mesons are taken only as pairs, as isospin singlets. In mean field approximation, we shall have the above conststruct giving isoscalar meson pairs with *even* parity. We aproximate the arbitrary number of meson pairs through the two functions, $f_M(\mathbf{r})$ and $f(\mathbf{r})$, the first indicating the distribution of mesons with respect to $\mathbf{x}$, and the second,the mutual correlation of the mesons.

We assume that $U^N_\alpha(\mathbf{r})$, $U^\Lambda_\alpha(\mathbf{r})$ and $f_M(\mathbf{r})$ are so normalised that

$$\int |U^N_\alpha(\mathbf{r})|^2 d\mathbf{r} = \int |U^\Lambda_\alpha(\mathbf{r})|^2 d\mathbf{r} = \int |f_M(\mathbf{r})|^2 d\mathbf{r} = 1.$$

As earlier [11], we take

$$U^N_\alpha(\mathbf{r}) = (\pi R_N^2)^{-3/4} e^{-\frac{r^2}{2R_N^2}}, \tag{9a}$$

$$U^\Lambda_\alpha(\mathbf{r}) = (\pi R_\Lambda^2)^{-3/4} e^{-\frac{r^2}{2R_\Lambda^2}}, \tag{9b}$$

$$f_M(\mathbf{r}) = (\pi R_M^2)^{-3/4} e^{-\frac{r^2}{2R_M^2}}, \tag{9c}$$

and

$$f(\mathbf{r}) = a(\pi R_\pi^2)^{-3/4} e^{-\frac{r^2}{2R_\pi^2}}, \tag{9d}$$

where a, R_M, R_π, R_N and R_Λ are arbitrary parameters which are determined variationally.

With the above construction, we shall now define the state of the system with n_N nucleons and n_Λ hyperons located at $\mathbf{x}$ as

$$|{}^AHe(\mathbf{x})> = N_R(\pi R_N^2)^{-3/4} \mathcal{N}_S(\mathbf{x})^\dagger \mathcal{M}(\mathbf{x})^\dagger |vac> \tag{10}$$

where N_R is the normalization constant, $A = n_N + n_\Lambda$ is the total number of baryons. We construct translationally invariant state of momentum $\mathbf{p}$ as [15]

$$|{}^AHe(\mathbf{p})> = N_R(2\pi)^{-3/2}(\pi R_N^2)^{-3/4} \int d\mathbf{x} e^{i\mathbf{p}.\mathbf{x}} \mathcal{N}_S(\mathbf{x})^\dagger \mathcal{M}(\mathbf{x})^\dagger \mid vac> . \tag{11}$$

The details regarding the evaluation of the normalisation constant are given in Appendix A. The normalisation constant is given by

$$N_R^{-2} = S_0 \tag{12}$$

with

$$S_0 = \sum_{n=0}^{\infty} \left(\frac{3a^2}{2}\right)^n \frac{1}{n!} \left(\frac{1}{a_n}\right)^{3/2}$$

where

$$a_n = \frac{n_N}{4} + \frac{n_\Lambda R_N^2}{4R_\Lambda^2} + \frac{nR_N^2}{4R_M^2}. \tag{13}$$

Then we shall proceed to evaluate the energy expectation values for the given system *at rest*.

III. ENERGY EXPECTATION VALUES AND ITS EXTREMIZATION

With $\mathcal{H}(\mathbf{z})$ as in equation (2) the energy operator is

$$H = \int \mathcal{H}(\mathbf{z}) d\mathbf{z}. \tag{14}$$

We are thus to find the expressions corresponding to equations (2) to obtain the energy. The advantage of the present approach is that with the ansatz function for the pions, we include an arbitrary number of pion pairs in a coherent manner with equal time algebra, which makes the contributions calculable while retaining basically nonperturbative 'higher order' effects without any truncation. Clearly, at present we are concentrating on the relevant attractive channel corresponding to σ-meson earlier [13] without using the above unphysical "particle". The picture is also completely quantum mechanical instead of classical.

A. Nucleon and hyperon kinetic energy

We shall now calculate the energy expectation values for the above configurations. We proceed to evaluate the expectation value of H_N. The expectation h_N for the same is defined through the equation, for $\mathbf{p} = 0$,

$$<^A He(\mathbf{p}')|H_N|^A He(\mathbf{p}) >= h_N \delta(\mathbf{p}' - \mathbf{p}). \tag{15}$$

In the above, we ignore the mass term. Then substitution of equation (3a) and (11) gives that

$$h_N = N_R^2 (\pi R_N^2)^{-3/2} \int g_N^{K.E}(\mathbf{r}) g_M(\mathbf{r}) d\mathbf{r}, \tag{16}$$

where

$$\begin{aligned} g_N^{K.E}(\mathbf{x}' - \mathbf{x}) &= \sum_{\alpha=1}^{N_n} \int U_\alpha^N(\mathbf{x}' - \mathbf{x}_\alpha)^* \left(-\frac{\nabla_{x_\alpha}^2}{2M_N} \right) U_\alpha^N(\mathbf{x} - \mathbf{x}_\alpha) d\mathbf{x}_\alpha \prod_{\beta \neq \alpha} \rho_\beta^N(\mathbf{x}' - \mathbf{x}) \prod_{\gamma=1}^{n_\Lambda} \rho_\gamma^\Lambda(\mathbf{x}' - \mathbf{x}) \\ &= \sum_{\alpha=1}^{n_N} \left(-\frac{\nabla_x^2}{2M_N} \right) \rho_\alpha^N(\mathbf{x}' - \mathbf{x}) \prod_{\beta \neq \alpha} \rho_\beta^N(\mathbf{x}' - \mathbf{x}) \prod_{\gamma=1}^{n_\Lambda} \rho_\gamma^\Lambda(\mathbf{x}' - \mathbf{x}). \end{aligned} \tag{17}$$

Substituting equations(9cand (A4) we then obtain from equation (17) as

$$g_N^{K.E}(\mathbf{r}) = \left[\frac{3}{M_N R_N^2} - \frac{\mathbf{r}^2}{2M_N R_N^4} \right] \exp\left(-\frac{n_N \mathbf{r}^2}{4R_N^2} \right). \tag{18}$$

Therefore equation (16) becomes

$$h_N = \frac{n_N}{4} \frac{3}{M_N R_N^2} \left(1 - \frac{S_1}{4S_0} \right), \tag{19}$$

where

$$S_1 = \sum_n \frac{1}{n!} \left(\frac{3a^2}{2} \right)^n \left(\frac{1}{a_n} \right)^{5/2}. \tag{20}$$

Similarly the kinetic energy for the hyperon is given as

$$h_\Lambda = \frac{n_\Lambda}{4} \frac{3}{M_\Lambda R_\Lambda^2} \left(1 - \frac{S_1}{4S_0} \right). \tag{21}$$

B. Meson kinetic energy

With the meson field operator expressions as in equation (4) we may write equation (3c) as

$$\mathcal{H}_M(\mathbf{z}) = a_i(\mathbf{z})^\dagger \omega_z a_i(\mathbf{z}). \tag{22}$$

The meson kinetic energy can be calculated through the equation

$$<^A He(\mathbf{p'})|H_M|^A He(\mathbf{p}) >= h_M \delta(\mathbf{p'}-\mathbf{p}). \tag{23}$$

Proceeding exactly in the same way as in ref. [11] we get

$$h_M = \sum_n (\frac{3a^2}{2})^n \frac{1}{n!} h_M^{(n)}, \tag{24}$$

where

$$h_M^{(n)} = \frac{3A^2}{S_0}\left(\frac{1}{\pi}\frac{4R_M^2 R_\pi^2}{a_n(4R_M^2+R_\pi^2)+R_N^2}\right)^{3/2} \int \exp\left[-\frac{4R_M^2 R_\pi^2 a_{n+1}}{a_n(4R_M^2+R_\pi^2)+R_N^2}\mathbf{q}^2\right]\omega(\mathbf{q})d\mathbf{q}. \tag{25}$$

C. Interaction energy

We now calculate the interaction energy. For this purpose we first note that

$$:\phi_i(\mathbf{z})\phi_i(\mathbf{z}) := \phi_i^{cr}(\mathbf{z})\phi_i^{cr}(\mathbf{z}) + \phi_i^{an}(\mathbf{z})\phi_i^{an}(\mathbf{z}) + 2\phi_i^{cr}(\mathbf{z})\phi_i^{an}(\mathbf{z}), \tag{26}$$

where we have substituted e.g.

$$\phi_i^{cr}(\mathbf{z}) = \frac{1}{\sqrt{2\omega_z}} a_i(\mathbf{z})^\dagger. \tag{27}$$

The expectation h_I for the same is defined through the equation, for $\mathbf{p}=0$ as

$$<^A He(\mathbf{p'})|H_I|^A He(\mathbf{p}) >= h_I \delta(\mathbf{p'}-\mathbf{p}). \tag{28}$$

Substitution of equation (3d) and (11) we have

$$\begin{aligned} & h_I\delta(\mathbf{p'}-\mathbf{p}) = \\ & N_R^2(\pi R_N^2)^{-3/2}\left[\frac{G_{\pi NN}^2}{2M_N}\int g_N^{INT}(\mathbf{x'}-\mathbf{z},\mathbf{x}-\mathbf{z}) g_M^{INT}(\mathbf{x'}-\mathbf{z},\mathbf{x}-\mathbf{z}) d\mathbf{z} e^{-i\mathbf{p'}.\mathbf{x'}+i\mathbf{p}.\mathbf{x}} d\mathbf{x'}d\mathbf{x}\right. \\ + & \left.\frac{G_{\pi\Lambda\Lambda}^2}{2M_\Lambda}\int g_\Lambda^{INT}(\mathbf{x'}-\mathbf{z},\mathbf{x}-\mathbf{z}) g_M^{INT}(\mathbf{x'}-\mathbf{z},\mathbf{x}-\mathbf{z}) d\mathbf{z} e^{-i\mathbf{p'}.\mathbf{x'}+i\mathbf{p}.\mathbf{x}} d\mathbf{x'}d\mathbf{x}\right] \end{aligned} \tag{29}$$

where

$$g_N^{INT}(\mathbf{x'}-\mathbf{z},\mathbf{x}-\mathbf{z}) =< vac|\mathcal{N}_\mathcal{S}(\mathbf{x'}) N_\alpha(\mathbf{z})^\dagger N_\alpha(\mathbf{z}) \mathcal{N}_\mathcal{S}(\mathbf{x})^\dagger|vac>,$$

$$g_\Lambda^{INT}(\mathbf{x'}-\mathbf{z},\mathbf{x}-\mathbf{z}) =< vac|\mathcal{N}_\mathcal{S}(\mathbf{x'}) \Lambda_\alpha(\mathbf{z})^\dagger \Lambda_\alpha(\mathbf{z}) \mathcal{N}_\mathcal{S}(\mathbf{x})^\dagger|vac>$$

and

$$g_M^{INT}(\mathbf{x'}-\mathbf{z},\mathbf{x}-\mathbf{z}) =< vac|\mathcal{M}(\mathbf{x'})\phi_i(\mathbf{z})\phi_i(\mathbf{z})\mathcal{M}(\mathbf{x})^\dagger|vac>$$

We first consider the cr-cr term of equation (26). Noting that

$$\begin{aligned} & < vac|\mathcal{N}_\mathcal{S}(\mathbf{x'}) N_\alpha(\mathbf{z})^\dagger N_\alpha(\mathbf{z}) \mathcal{N}_\mathcal{S}(\mathbf{x})^\dagger|vac> \\ & = n_N \exp\left(-\frac{(n_N-1)\mathbf{r}^2}{4R_N^2} - \frac{n_\Lambda \mathbf{r}^2}{4R_\Lambda^2}\right) U^N(\mathbf{x'}-\mathbf{z})^* U^N(\mathbf{x}-\mathbf{z}), \end{aligned} \tag{30}$$

$$< vac|\mathcal{N}_{\mathcal{S}}(\mathbf{x}')\Lambda_\beta(\mathbf{z})^\dagger\Lambda_\beta(\mathbf{z})\mathcal{N}_{\mathcal{S}}(\mathbf{x})^\dagger|vac >$$
$$= n_\Lambda \exp\left(-\frac{(n_\Lambda-1)\mathbf{r}^2}{4R_\Lambda^2} - \frac{n_N\mathbf{r}^2}{4R_N^2}\right) U^\Lambda(\mathbf{x}'-\mathbf{z})^* U^\Lambda(\mathbf{x}-\mathbf{z}) \tag{31}$$

and

$$< vac|\mathcal{M}(\mathbf{x}')\phi_i^{cr}(\mathbf{z})\phi_i^{cr}(\mathbf{z})\mathcal{M}(\mathbf{x})^\dagger|vac >= g_M(\mathbf{x}'-\mathbf{x}) < vac|B(\mathbf{x}')\phi_i^{cr}(\mathbf{z})\phi_i^{cr}(\mathbf{z})|vac > . \tag{32}$$

We get the contribution to energy from the cr-cr term from equation (26) as

$$h_I^{crcr} = \sum_n (\frac{3a^2}{2})^n \frac{1}{n!} h_I^{crcr(n)} \tag{33}$$

where

$$h_I^{crcr(n)} = \frac{3a}{\pi S_0}\left(\frac{R_M^2 R_\pi^2}{\pi^2}\right)^{3/4} a_n^{-3/2} \int Q^2 dQ q^2 dq \int_0^1 dx [W(Q^2,q^2,x^2)]^{-1/4}$$
$$\left(\frac{n_N G_{\pi NN}^2}{M_N} \exp(-\frac{q^2 R_N^2}{4}) + \frac{n_\Lambda G_{\pi\Lambda\Lambda}^2}{M_\Lambda} \exp(-\frac{q^2 R_\Lambda^2}{4})\right)$$
$$\exp\left[-(\frac{R_N^2}{16a_n} + \frac{R_M^2}{2})q^2 - \frac{R_\pi^2 Q^2}{2}\right], \tag{34}$$

and

$$[W(Q^2,q^2,x^2)] = [(\mu^2 + Q^2 + \frac{1}{4}q^2)^2 - Q^2 q^2 x^2].$$

Similarly, the expression for the anhilation-anhilation term of equation (26) can be calculated. This expression is exactly identical to the above expression (34). The energy expectation value for creation-annihilation term of equation (26) can be written exactly in the similar way as

$$h_I^{cran} = \sum_n h_I^{cran(n)} (\frac{3A^2}{2})^n \frac{1}{n!} \tag{35}$$

where

$$h_I^{cran(n)} = \frac{3a^2\ G^2}{8\pi^4\ S_0}\left(\frac{4R_M^2 R_\pi^2}{\pi a_n(4R_M^2 + R_\pi^2) + R_N^2}\right)^{3/2} \int Q^2 dQ q^2 dq \int_0^1 dx [W(Q^2,q^2,x^2)]^{-1/4}$$
$$\left(\frac{n_N}{M_N} \exp(-\frac{q^2 R_N^2}{4}) + \frac{n_\Lambda}{M_\Lambda} \exp(-\frac{q^2 R_\Lambda^2}{4})\right)$$
$$\exp\left[-(\frac{4R_M^2 + R_\pi^2}{16})\mathbf{q}^2 - \frac{4R_M^2 R_\pi^2 a_{n+1} Q^2}{a_n(4R_M^2 + R_\pi^2) + R_N^2}\right]. \tag{36}$$

Adding equation (33) and (35) we have the interaction energy as

$$h_I = 2(h_I^{crcr} + h_I^{cran}) \tag{37}$$

D. Baryon repulsion energy

We next have to include the energy of repulsion which may arise from ω-exchange [10,11]. The Hamiltinian is given by the simple form

$$H_R \approx \frac{g_{\omega NN}^2}{2m_\omega^2}\int N_\alpha(\mathbf{z})^\dagger N_\alpha(\mathbf{z}) N_\beta(\mathbf{z})^\dagger N_\beta(\mathbf{z}) d\mathbf{z} + 2\times\frac{g_{\omega NN}\ g_{\omega\Lambda\Lambda}}{2m_\omega^2}\int N_\alpha(\mathbf{z})^\dagger N_\alpha(\mathbf{z})\Lambda_\beta(\mathbf{z})^\dagger\Lambda_\beta(\mathbf{z})d\mathbf{z}$$
$$+ \frac{g_{\omega\Lambda\Lambda}^2}{2m_\omega^2}\int \Lambda_\alpha(\mathbf{z})^\dagger\Lambda_\alpha(\mathbf{z})\Lambda_\beta(\mathbf{z})^\dagger\Lambda_\beta(\mathbf{z})d\mathbf{z} \tag{38}$$

where m_ω, $g_{\omega NN}$ and $g_{\omega\Lambda\Lambda}$ are ω-meson mass, ω - N coupling and $\omega - \Lambda$ coupling respectively. Now we calculate the energy due to repulsion through the equation

$$<^A He(\mathbf{p'})|H_R|^A He(\mathbf{p})>= h_R \delta(\mathbf{p'}-\mathbf{p}). \tag{39}$$

After a little calculation we get

$$\begin{aligned} h_R = & n_N(n_N-1)\frac{g^2_{\omega NN}}{2{m_\omega}^2}\left(\frac{1}{2\pi R_N^2}\right)^{3/2} + 2n_N n_\Lambda \frac{g_{\omega NN}\ g_{\omega\Lambda\Lambda}}{2{m_\omega}^2}\left(\frac{1}{\pi(R_N^2+R_\Lambda^2)}\right)^{3/2} \\ & + n_\Lambda(n_\Lambda-1)\frac{g^2_{\omega\Lambda\Lambda}}{2{m_\omega}^2}\left(\frac{1}{2\pi R_\Lambda^2}\right)^{3/2}. \end{aligned} \tag{40}$$

E. Coulomb repulsion energy

We next have to include the coulomb repulsion energy. The Hamiltonian is given by the form

$$H_C = \alpha \int N_{p_{1/2}}(\mathbf{Z}+\frac{\mathbf{z}}{2})^\dagger N_{p_{1/2}}(\mathbf{Z}+\frac{\mathbf{z}}{2}) N_{p_{-1/2}}(\mathbf{Z}-\frac{\mathbf{z}}{2})^\dagger N_{p_{-1/2}}(\mathbf{Z}-\frac{\mathbf{z}}{2}) \mid z \mid^{-1} d\mathbf{Z}d\mathbf{z} \tag{41}$$

where $\alpha = 1/137$. The coulomb energy is calculated through the equation

$$<^A He(\mathbf{p'}) \mid H_C|^A He(\mathbf{p})>= h_C \delta(\mathbf{p'}-\mathbf{p}). \tag{42}$$

Using equation (11) and (41) we have from equation (42) as

$$h_C = \frac{1}{S_0}\alpha(\pi R_N^2)^{-3/2}\int \exp(-\frac{\mathbf{r}^2}{2R_N^2}) g_M(\mathbf{r}) h_c(\mathbf{r},\mathbf{z}) d\mathbf{r}d\mathbf{z}, \tag{43}$$

where

$$h_c(\mathbf{r},\mathbf{z}) = \int U_\alpha^N(\mathbf{x'}-\mathbf{Z}-\frac{\mathbf{z}}{2})^* U_\alpha^N(\mathbf{x}-\mathbf{Z}-\frac{\mathbf{z}}{2}) U_\alpha^N(\mathbf{x'}-\mathbf{Z}+\frac{\mathbf{z}}{2}) U_\alpha^N(\mathbf{x}-\mathbf{Z}+\frac{\mathbf{z}}{2})^* \frac{1}{|z|} d\mathbf{Z} \tag{44}$$

After little algebra, we have

$$h_C = \left(\frac{8}{\pi}\right)^{1/2}\frac{\alpha}{R_N}. \tag{45}$$

F. Meson repulsion energy

Further, we have taken the pions to be point like, and assumed that they can approach as near as possible, which is physically not correct. If we bring two pions close to each other there will be an effective force of repulsion because of their composite structure. We shall now assume a phenomenological term corresponding to the meson repulsion as [10,11]

$$h_M^R = \sum_n {h_M^R}^{(n)} (\frac{3A^2}{2})^n \frac{1}{n!}, \tag{46}$$

where

$${h_M^R}^{(n)} = \frac{3A^2}{S_0}\left(\frac{4R_M^2 R_\pi^2}{\pi a_n(4R_M^2+R_\pi^2)+R_N^2}\right)^{3/2} 0.65 \int \exp\left[-\frac{4R_M^2R_\pi^2 a_{n+1}}{a_n(4R_M^2+R_\pi^2)+R_N^2}\mathbf{q}^2\right] e^{R_c^2 q^2} d\mathbf{q} \tag{47}$$

with $R_c^2 = 1.2$ fm^2.

We have now completed the frame work for dressing of hypernuclei with scalar and iso-scalar pion pairs. We have also discussed the baryon repulsion, meson repulsion and coulomb repulsion. With all these contributions, the situation for nuclear structure can be realistic. We next minimise the energy given by

$$E = h_N + h_M + h_I + h_R + h_C + h_M^R \tag{48}$$

with the parameters $a,\ R_M,\ R_\pi,\ R_N$.

IV. RESULTS AND DISCUSSIONS

In our study we calculate the binding energy using a nonperturbative method. The nucleon wavefunction and the function for pion dressing are determined by minimising the energy E given by (48). The masses of different particles and the coupling constants used in the calculation are given in Table 1. The pion-nucleon and ω-nucleon coupling constants are taken from ref. [10]. Following Greiner [1], we have taken $\omega - \Lambda$ coupling constant to be $\frac{2}{3}$ of $\omega - N$ coupling constant. This relation has been obtained from SU(6). The pion-hyperon coupling constant is taken to be 14. The $\pi - \Sigma$ hyperon and $\omega - \Sigma$ hyperon coupling constants are taken to be the same as those of $\pi - \Lambda$ and $\omega - \Lambda$.

In our calculation, we include the binding energy of ^{4}He by extremising the total energy E given by eq. (48) as a check for our programe. The binding energy for this nucleus is found to be –28.96 MeV which is the same as earlier value [11] and agrees quite well with experiment. The variational parameters $a,\ R_M,\ R_\pi,\ R_N$ obtained from energy minimization are given in Table 2. Then we added hyperons to this nucleus and repeat the calculation. For $^{4+n\Lambda}$He and $^{4+n\Sigma}$He, the parameters $a,\ R_M,\ R_\pi,\ R_N$ are taken same as that for ^{4}He. This is because $a,\ R_M,\ R_\pi$ corresponds to meson space and hence are not likely to change. Again since the number of nucleons remain same, R_N is also kept fixed for $^{4+n\Lambda}$He and $^{4+n\Sigma}$He. In the calculation of $^{4+n\Lambda}$He, R_Λ is the only parameter which is obtained from energy minimization. Similarly for $^{4+n\Sigma}$He, R_Σ is the only variational parameter which is determined from the minimization of energy. We may suspect that the presence of hyperon will change the earlier parameters, but a free variation of all the parameters like a, R_M, R_π, R_N and R_Λ or R_Σ in each case in the energy minimisation has negligible effect on the final result. The different variational parameters are given in Table 2. An analysis of Table 2 indicates that the variational parameter R_Λ or R_Σ increases as we go to double hypernuclei. This shows that there developes a Λ- or Σ-halo around the nucleus. Greiner [1] obtained a similar result for multi-hyper nuclei with their RMF calculation. Since there exists hyperon halos around the nucleus, it is likely that the hyperons will dissociate under peripheral collisions. This may have observational effects.

We then performed a similar calculation for ^{3}He and determined its binding energy by extremizing the total energy E given by equation (48). From our calculation, we find the binding energy for this nucleus to be –7.09 MeV which agrees quite well with experimental value. The corresponding variational parameters $a,\ R_M,\ R_\pi,\ R_N$ are given in Table 2. The values of these parameters are kept constant for multi-hyperon systems based on ^{3}He like $^{3+n\Lambda}$He and $^{3+n\Sigma}$He. For $^{3+n\Lambda}$He, the only variational parameter is R_Λ and R_Σ is the only variational parameter for $^{3+n\Sigma}$He. For these multihypernuclei also, we observe Λ-halos and Σ- halos as in the previous case.

The binding energies of different multi-lambda hypernuclei were calculated by Greiner [1]. Their results agree quite well with our calculated results for $^{4+n\Lambda}$He. We also find that the binding energies increase when Λ-hyperons are added to normal nuclei. However for Σ-hyperons, the increase in binding energy is quite small.

Then we have calculated the hyperon separation energies for different nuclei. The Λ and Σ separation energies are given as

$$B_\Lambda = B({}_\Lambda A) - B(A-1)$$

$$B_\Sigma = B({}_\Sigma A) - B(A-1)$$

Here ${}_{\Lambda/\Sigma}A$ corresponds to a nucleus with $(A-1)$ nucleons and a hyperon. Similarly the $\Lambda\Lambda$ and $\Sigma\Sigma$ separation energies are given by

$$B_{\Lambda\Lambda} = B({}_{\Lambda\Lambda}A) - B(A-2)$$

$$B_{\Sigma\Sigma} = B({}_{\Sigma\Sigma}A) - B(A-2)$$

where ${}_{\Lambda\Lambda/\Sigma\Sigma}A$ stands for total number of nucleons and hyperons. The separation energies are given in Table 3.

From Table 3, we find that the hyperon separation energy is reasonably well reproduced for $^{4+\Lambda}$He and $^{3+\Sigma}$He. However, the separation energy for $^{3+\Lambda}$He is off from experiment by a factor of 2. The experimental data for double hyperon separation energy exists only in $^{4+2\Lambda}$He. The experimental value is 10.6 MeV. From our calculation, we find this value to be 3.5 MeV. Greiner [1] has also obtained a similar result for this nucleus. However, they have observed that one should include additional hyperon-hyperon interaction generated by the exchange of mesons with hidden strangeness. They have correctly reproduced the double hyperon separation energy within this frame-work. We also plan to carry out a similar calculation for the case of multi-hyperon nuclei by including strange mesons.

V. CONCLUSION

We have developed a nonperturbative technique to study the binding energies of s-state hyper nuclei. This model corresponds to the quantum picture of classical fields of Walecka in which σ-meson effects are simulated through coherent state of pion pairs. Here all the parameters are calculated variationally through energy minimization.

We have calculated the binding energies of ^{4}He and ^{3}He by minimisation of energy. The agreement with experiment is quite satisfactory. The binding energies of the s-state hypernuclei are calculated by energy minimisation with respect to R_Λ or R_Σ. The pion and nucleon parameters are not varied but taken same as those for the neighbouring nucleus. As lambda-hyperons are added to the nucleus, the binding energies of the resulting hypernuclei are found to increase. However, the increase in binding energy is quite small when Σ hyperons are added. We find that hypernuclei develop Λ- and Σ-halos. Similar results are also obtained by Greiner from his RMF calculation. Since Λ- or Σ- halos develop in these nuclei, the hyperons may dissociate under peripheral collisions which may have observationaleffects. We have calculated the hyperon separation energies. The agreeement with experiment for hyperon separation energy is reasonable. The double hyperon separation energies are underestimated by a factor around of three. Greiner has suggested that one should use additional interaction to reproduce the correct double hyperon separation energy.

Thus without using a nuclear potential and without any adjustable parameters, we have been able to reproduce satisfactorily the binding energies of ^{3}He, ^{4}He and the corresponding s-state hypernuclei.

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APPENDIX A:

We normalize the state through

$$<^A He(\mathbf{p}')|^A He(\mathbf{p})>=\delta(\mathbf{p}'-\mathbf{p}) \tag{A1}$$

which will determine the normalization constant N_R as shown below. Let us define

$$g_{N\Lambda}(\mathbf{x}'-\mathbf{x})=<vac|\mathcal{N}_S(\mathbf{x}')\mathcal{N}_S(\mathbf{x})^\dagger|vac>\ . \tag{A2}$$

We have anticipated that the above expression is a function of $(\mathbf{x}'-\mathbf{x})$. In fact, a direct evaluation of yields

$$g_{N\Lambda}(\mathbf{x}'-\mathbf{x})=\prod_{\alpha=1}^{n_N}\int U_\alpha^N(\mathbf{x}'-\mathbf{x}_\alpha)^*U_\alpha^N(\mathbf{x}-\mathbf{x}_\alpha)d\mathbf{x}_\alpha\prod_{\beta=1}^{n_\Lambda}\int U_\beta^\Lambda(\mathbf{x}'-\mathbf{x}_\beta)^*U_\beta^\Lambda(\mathbf{x}-\mathbf{x}_\beta)d\mathbf{x}_\beta. \tag{A3}$$

It will be useful to define

$$\rho_\alpha^N(\mathbf{x}'-\mathbf{x})=\int U_\alpha^N(\mathbf{x}'-\mathbf{x}_\alpha)^*U_\alpha^N(\mathbf{x}-\mathbf{x}_\alpha)d\mathbf{x}_\alpha, \tag{A4}$$

and

$$\rho_\alpha^\Lambda(\mathbf{x}'-\mathbf{x})=\int U_\alpha^\Lambda(\mathbf{x}'-\mathbf{x}_\alpha)^*U_\alpha^\Lambda(\mathbf{x}-\mathbf{x}_\alpha)d\mathbf{x}_\alpha. \tag{A5}$$

Therefore we have

$$g_{N\Lambda}(\mathbf{x}'-\mathbf{x})=\prod_{\alpha=1}^{n_N}\rho_\alpha^N(\mathbf{x}'-\mathbf{x})\prod_{\beta=1}^{n_\Lambda}\rho_\beta^\Lambda(\mathbf{x}'-\mathbf{x})\equiv g_N(\mathbf{x}'-\mathbf{x})g_\Lambda(\mathbf{x}'-\mathbf{x}). \tag{A6}$$

Similarly we define

$$g_M(\mathbf{x}'-\mathbf{x})=<vac|\mathcal{M}(\mathbf{x}')\mathcal{M}(\mathbf{x})^\dagger|vac>\ . \tag{A7}$$

The above equation is evaluated in the "mean field aproximation". In fact,

$$< vac|\mathcal{M}(\mathbf{x}')\mathcal{M}(\mathbf{x})^\dagger \mid vac >= \sum_{n=0}^{\infty} \frac{1}{(n!)^2} < vac \mid (B(\mathbf{x}'))^n (B(\mathbf{x})^\dagger)^n |vac> . \tag{A8}$$

In the above we aproximate

$$< vac|(B(\mathbf{x}'))^n (B(\mathbf{x})^\dagger)^n |vac >= n! \left[< vac|B(\mathbf{x}^{'})B(\mathbf{x})^\dagger |vac>\right]^n \tag{A9}$$

i.e. we replace products of $B(\mathbf{x}')B(\mathbf{x})^\dagger$ by their vacuum expectation values, given as

$$\begin{aligned} &< vac|B(\mathbf{x}')B(\mathbf{x})^\dagger |vac> \\ &= \frac{3}{2} \int f_M \left(\mathbf{x}' - \frac{\mathbf{z}_1 + \mathbf{z}_2}{2}\right)^* f_M \left(\mathbf{x} - \frac{\mathbf{z}_1 + \mathbf{z}_2}{2}\right) f(\mathbf{z}_1 - \mathbf{z}_2)^* f(\mathbf{z}_1 - \mathbf{z}_2) d\mathbf{z}_1 d\mathbf{z}_2. \end{aligned} \tag{A10}$$

We then obtain from equation (A1), (A2) and (A7) that

$$<^A He(\mathbf{p}')|^A He(\mathbf{p}) >= (2\pi)^{-3} N_R^2 (\pi R_N^2)^{-3/2} \int g_{N\Lambda}(\mathbf{x}' - \mathbf{x}) g_M(\mathbf{x}' - \mathbf{x}) e^{(-i\mathbf{p}'.\mathbf{x}' + i\mathbf{p}.\mathbf{x})} d\mathbf{x}' d\mathbf{x}. \tag{A11}$$

Substituting $(\mathbf{x}' + \mathbf{x})/2 = \mathbf{X}$ and $(\mathbf{x}' - \mathbf{x}) = \mathbf{r}$ and comparing with equation (A1) we then obtain from (A11) that, for $\mathbf{p} = 0$,

$$N_R^2 (\pi R_N^2)^{-3/2} \int g_{N\Lambda}(\mathbf{r}) g_M(\mathbf{r}) d\mathbf{r} = 1, \tag{A12}$$

We obtain from equation (9a), (9b), (9c) and (9d) that

$$g_N(\mathbf{r}) = \exp\left(-\frac{n_N \mathbf{r}^2}{4R_N^2}\right) \tag{A13a}$$

$$g_\Lambda(\mathbf{r}) = \exp\left(-\frac{n_\Lambda \mathbf{r}^2}{4R_\Lambda^2}\right) \tag{A13b}$$

and

$$g_M(\mathbf{r}) = \exp\left(\frac{3}{2} a^2 \exp\left(-\frac{\mathbf{r}^2}{4R_M^2}\right)\right). \tag{A13c}$$

Hence, by equation (A12), N_R^{-2} depends nonlinearly on the parameter a, R_N and R_M and infact we explicitly have

$$N_R^{-2} = (\pi R_N^2)^{-3/2} \sum_{n=0}^{\infty} \left(\frac{3a^2}{2}\right)^n \frac{1}{n!} \int \exp\left[-\frac{n_N \mathbf{r}^2}{4R_N^2} - \frac{n_\Lambda \mathbf{r}^2}{4R_\Lambda^2} - \frac{n\mathbf{r}^2}{4R_M^2}\right] d\mathbf{r}. \tag{A14}$$

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Table 1

M_N	M_Λ	M_Σ	μ	m_ω	$G^2_{\pi NN}/4\pi$	$g_{\omega NN}$	$G^2_{\pi\Lambda\Lambda}/4\pi$	$g_{\omega\Lambda\Lambda}$
940	1115.6	1192.6	140	780	14.6	4.2	14.0	$(2/3)g_{\omega NN}$

Table 2

Nucleus	Binding Energy MeV	a	R_M fm	R_π fm	R_N fm	R_Λ fm	R_Σ fm
^{4}He	–28.96	–0.329	1.151	1.696	1.64	–	–
$^{4+\Lambda}$He	–31.42	”	”	”	”	2.258	–
$^{4+2\Lambda}$He	–32.42	”	”	”	”	2.426	–
$^{4+\Sigma}$He	–29.92	”	”	”	”	–	2.605
$^{4+2\Sigma}$He	–29.84	”	”	”	”	–	2.87
^{3}He	–7.09	”	”	”	1.207	–	–
$^{3+\Lambda}$He	–12.30	”	”	”	”	2.282	–
$^{3+2\Lambda}$He	–15.8	”	”	”	”	2.4	–
$^{3+\Sigma}$He	–10.61	”	”	”	”	–	2.459
$^{3+2\Sigma}$He	–12.85	”	”	”	”	–	2.598

Table 3

Nucleus	Hyperon Separation Energy MeV	Experimental Value MeV
	B_{Λ}	
$^{4+\Lambda}$He	2.46	3.12
$^{3+\Lambda}$He	5.21	2.39
	B_{Σ}	
$^{4+\Sigma}$He	0.96	
$^{3+\Sigma}$He	3.52	3.2
	$B_{\Lambda\Lambda}$	
$^{4+2\Lambda}$He	3.46	10.6
$^{3+2\Lambda}$He	8.71	
	$B_{\Sigma\Sigma}$	
$^{4+2\Sigma}$He	0.88	
$^{3+2\Sigma}$He	5.76	

Table Captions

Table 1. Masses of nucleon (M_N), Λ-hyperon (M_Λ) Σ-hyperon (M_Σ) pion (μ) and ω-meson (m_ω) in MeV. Pion-nucleon, omega-nucleon, pion-lambda and omega-lambda coupling constants used in the calculation are also given.

Table 2. Binding energies and variational parameters obtained from energy minimisation are given for different hypernuclei.

Table 3. The single hyperon separation energy and double hyperon separation energy for different hypernuclei. The experimental data are taken from [16] and [17].